Cooperative and Non-Cooperative UAS Traffic Volumes

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***Abstract*—We describe an analytical process to determine how much UAS traffic is feasible. The process is a simulator and data processing tools. The two are applied to the US San Francisco Bay Area and Norrköping, Sweden. The amount of UAS traffic is measured in flights per day and simulated up to 200,000 flights. A UAS traffic volume is feasible if specified metrics meet operational requirements with high probability and are stable, in the sense of being below thresholds observed for monotone properties in random geometric graphs. We focus on conflict cluster size and argue for it as a fundamental safety metric worthy of extraordinary consideration.**

***Keywords*—capacity, airspace, UTM, unmanned**

I. Introduction

We are interested in an analytical process able to determine how much unmanned air traffic is feasible. We think of the amount of air traffic in this paper and prior work [1] as the number of UAS flights per day in a metropolitan region. Metropolitan regions are an appropriate geographical scale for major UAS use cases like package delivery or digital scanning services. We present analyses of the San Francisco Bay Area in the United States and the Norrköping region in Sweden, and consider up to 200,000 flights per day because prior analyses suggest 100,000 flights per day as possible for the Bay Area just for package delivery [1].

Feasibility in aviation is informed by safety, though there are other considerations as well. The analytical work here focuses only on safety. A preliminary application of our analytical process to noise capacity appears in [2].

The safety of unmanned aviation will be informed by the technology applied to it. This technology will have an unmanned traffic management (UTM) [3] component in the network core (the cloud), and a collision detection, resolution, and avoidance (CDRA) [4] component on the network edge, i.e., on-board each UAS. An analytical process aiming to establish the feasibility of a volume of unmanned air traffic needs to pick metrics for safety, parameters modeling the technology, and develop a computational process that puts numbers to the safety metrics, as a function of the technology parameters. Our computational process is a simulator described in section IV.

Safety metrics in the literature pertain to the number of aircraft conflicts, forced conflicts, average proximity, closest approach distances, and compounds of these such as dynamic density [5]–[7]. Some of these are extrinsic and others intrinsic [8].

Amongst these, we value the aircraft conflict cluster size measure proposed by Durand [9] and Bilimoria[10] more highly in the context of unmanned aviation. In a graph theoretic manner, we think of UAS as nodes, with an edge between any two UAS in conflict, for any suitable conflict definition (see figure 1). A conflict cluster is then a graph component and conflict cluster size is the number of vertices in the component. Cluster size statistics become component statistics in random geometric graphs [11]. This provides rich theoretical support for a metric sensible in aviation. In this paper we model two conflict definitions. One abstractly models non-cooperative CDRA and the other cooperative. Our analytical process then turns the two definitions into feasible volumes of unmanned air traffic for the San Francisco Bay Area and Norrköping. As one might guess, conflict cluster sizes turn out to be smaller for cooperative CDRA. Our prior work was entirely non-cooperative and for the Bay Area alone [1].

We propose the conflict cluster size metric be distinguished in unmanned aviation because as an architectural separator between UTM and on-board CDRA, i.e., the intelligence in the network core and that on the network edge. Architectural separations, when sensible, are enormously valuable, as they decouple industries and decompose system design problems. Conflict cluster size can separate UTM and CDRA because it determines the computational complexity of many CDRA algorithms [4,12]–[19]. The running time of these algorithms grows with the number of aircraft to be jointly de-conflicted, i.e. conflict cluster size.

While pilot ability or workload may be the dominant consideration in manned CDRA, computational complexity should be its surrogate for UAS CDRA. The separated design

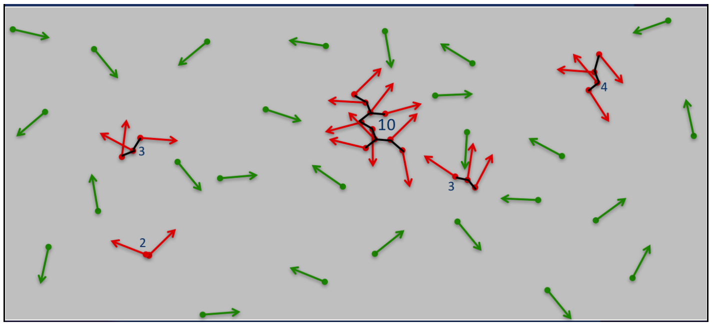

Fig. 1: Abstraction of an airspace snapshot as a graph with cluster sizes indicated. (Aircraft in conflict are shown by red dots. The conflict pairs are connected by a black line. The arrows indicate the heading of the aircraft.)

philosophy based on conflict cluster size is then as follows. The purpose of any UTM would be to ensure that on-board CDRA experiences problems of low complexity, i.e, only small conflict clusters, with high probability. In this view a UTM fails not only when there is a collision. A UTM should be audited and reviewed each time an on-board CDRA is compelled to resolve a conflict cluster of significant size.

This clarifies the basics of our analytical process. A volume of unmanned air traffic is feasible when it produces conflict clusters of significant size with low probability. Our task is now reduced to clarifying what size is significant and what probability is low.

An analysis of current air and road transportation suggests conflicts involving 3 or 4 UAS to be the appropriate limit for on-board CDRA. TCAS and ACAS-X performance both deteriorate significantly when faced with more than two vehicle conflicts [20]. The road transportation system is operated less conservatively, but even there, one observes a three vehicle limit. Drivers solve a three vehicle coordination problem in motion when merging into a freeway or changing lanes. Intersections can require coordination of more than three vehicles but they are brought to a halt with stop signs or traffic lights. Transportation systems almost universally limit on-board CDRA to negotiating at most 3 vehicles through a combination of structure and operational controls. Therefore we focus on the prevalence of conflicts clusters of size greater than 3. Our simulator and data processing tools quantify their probability of occurrence, number, and frequency.

Frequency is modeled by the fraction of journey time spent by a UAS in conflict with other UAS. We call this the Normalized Time spent in Conflict (NTSC). We take a mean of the individual NTSCś of all UAS in the system and assign it as the system NTSC. More precisely -

$$NTSC = \frac{Time\ in\ conflict\ with\ at\ least\ one\ other\ UAS}{Total\ transit\ duration\ of\ own\ UAS} \tag{1}$$

.

The issue of low probability is more complex. Large conflict cluster sizes are undesirable and must therefore be improbable. The assignment of a tolerable limit on the probability of large conflicts is more a policy exercise than a scientific one. However we argue that the proper operating regime of an unmanned aviation system is one in which the probability of the undesirable is not only low but also stable, i.e., small increases in the volume of air traffic should not entail large increases in the probability of occurrence of large conflicts. This point has scientific support because the probabilities associated with monotone properties in random geometric graphs, i.e, properties preserved by the addition of edges, exhibit rapid transitions about thresholds [11].

For example, one of the curves in figure 7, section V (the 300m curve), shows the probability of occurrence of a conflict cluster of size greater than 3, as being under 0.2 at 20,000 UAS/day. The same probability crosses 0.8 at 40,000 UAS/day. Thus we consider the unmanned traffic regime between 20,000 and 40,000 UAS flights per day a region of rapid transition, and interpret the curve to mean that the amount of UAS traffic in the metropolitan region under the assumptions of the curve should be kept below 20,000 flights per day.

More formally for any metric M that is a function of the amount of UAS traffic N, having an acceptable limit M', we define the η transition range $[N_l,\ N_r]$ for any $\eta \in (0, 0.5)$ if $\mathsf{P}\{M(N_l - 1) > M'\} < \eta$ and $\mathsf{P}\{M(N_r + 1) > M'\} > 1 - \eta$.

In the example above, the metric $M(N)$ is the size of the largest conflict cluster, N the number of flights per day, $M' = 3$, and $\eta = 0.2$.

The structure of the paper is as follows. We first present a review of related work that motivates this paper under section II. Our conflict definition and the difference between the conflict conditions for cooperative and non cooperative scenarios is discussed in III. Section IV presents and describes our simulation setup in detail. In V we discuss our estimates for Bay Area followed by a capacity comparison with Norrköping. The results also focus on the tails of the distributions or the 99^{th} percentile to emphasize the larger and more hazardous conflicts. The increase in capacity and reduction in complexity is promising. However, this requires further research to establish tighter bounds by modeling more airborne technologies. This is the subject of section VI which also summarizes the findings of the paper.

II. Literature Review

Future Unmanned Aircraft System (UAS) operations may be free flight by nature i.e. individual flights could prefer responsibility for determining their own courses independent of a global plan or system. UAS Traffic Management (UTM) should therefore support user preferred flight trajectories to the extent possible. Any chosen metrics should account for this.

Several ‘self-separation’ design concepts and decentralized control strategies that transfer some of the separation responsibility to the cockpit have been proposed for manned aviation [4,21,22]. Air traffic management (ATM) architectures with the same objective for manned free flight were also researched by Bilimoria et. al at NASA as part of their Distributed Air/Ground Traffic Management (DAG-TM) concept[21,23]–[25]. DAG-TM is characterized by distributed information sharing, decision-making and/or responsibility among a triad of agents: the Flight Deck (FD), Air Traffic Service Provider (ATSP), and Airline Operational Control (AOC). From a UTM perspective, this is analogous to the on board autopilot (FD), the UTM service provider (ATSP) and the UAS operator/command center (AOC).

[24] shows that four types of metrics can be potentially used to evaluate any UTM architecture for free flight (the sample measures used in [24] for manned ATM are listed in parenthesis) - Performance (Change in direct operating cost), Safety (number of actual conflicts and conflict alerts), Stability (number of forced conflicts (domino effect)) and Dynamic Density (DD) (aircraft density, average proximity and average point of closest approach). In this work, we focus on safety.

The capacity of an airspace can be fundamentally understood as the maximum number of aircraft that it can safely accommodate. Capacity estimation approaches in literature evaluate this safety from controller and pilot workload under a given set of constraints [26]–[29]. Conventional air traffic complexity measures such as Monitor Alert Parameter (MAP), the maximum number of aircraft an air traffic control (ATC) controller can handle at any given time and DD, a weighted sum of factors that affect the air traffic complexity, are defined based on an assumption of a structured airspace and ATM that includes monitors, sectors and airways. Most of this structure doesn't exist in low-altitude airspace where the future UAS traffic is expected to show up. Moreover as unmanned aviation begets the need for automation, these extrinsic measures may need to be modified for automated traffic.

A second more intrinsic approach (in the sense of Vidosavljevic [8]) for UAS traffic is presented in [1]. The safety of future UAS traffic is measured from the expected conflict cluster size statistics derived from an aircraft cluster based analysis as defined and discussed by Durand[9] and Bilimoria[10]. On board systems must resolve these conflicts as the UAS fly. Multi-aircraft de-confliction algorithmic research [4,12]–[19], parametrizes the computational complexity of conflict resolution on the number of aircraft to be jointly de-conflicted. Hence, cluster size makes computational complexity a function of air traffic complexity.

We need to account for the presence or absence of cooperation between the UAS. This is incorporated in the conflict conditions. In aviation, conflict scenarios can be either tactical(close range, immediate action, e.g.-TCAS[30]) or strategic (long-range, can be smoothly resolved). A simple example of a strategic maneuver as used by Bilimoria et. al. [24] is shown in Figure 2. Either of the scenarios can have cooperative or non-cooperative aircraft. Cooperative aircraft maneuver to assist in avoiding the conflict and therefore require some form of information exchange on intent and protocol. A non cooperative aircraft on the other hand could do anything including heading straight for the other aircraft at full speed. There is no intent exchange and hence this mimics a lack of airborne awareness. We therefore model cooperative aircraft to be in conflict only if they are within a minimum distance and *converging* while non cooperative aircraft to be in conflict if they are just within the minimum distance, even

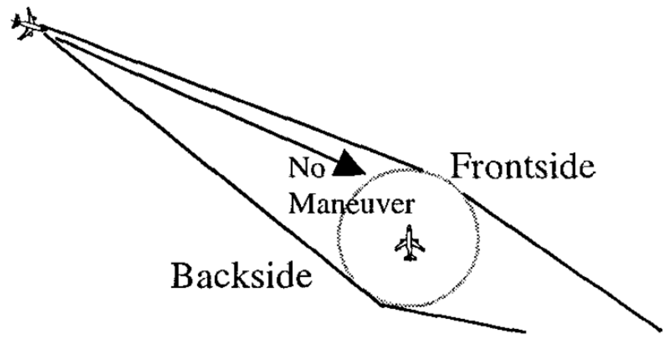

Fig. 2: CD&R geometry based on choosing the lower cost choice between the frontside and backside maneuver [24]

if they are *diverging*. In section III, we describe the conflict detection conditions in further detail.

Lastly, ATM and its simulators are focused mostly on scheduled and deterministic traffic. Aircraft take off and land in distinct and well defined areas. With respect to UAS, this will change completely in future when, in the words of Dr. Kopardekar [31],"every home will have a drone and every home will serve as an aerodrome". Further, given the diversity and unpredictability (temporal and spatial) of its operations, randomness is a much bigger player for future UAS traffic. To account for this stochastic component, we look at some past approaches. In particular, the probabilistic setup which can be called *Dutch model* was used in PhD thesis of Hoekstra [32], developed by Jardin [33], and more recently explored within the Metropolis project by TU Delft [34]. In this model the aircraft are distributed *uniformly* in the given airspace. In the basic version of the Dutch model, the direction of flight is also uniformly distributed in 0...360°; in [34] the different direction cones are separated by altitude. This uniform spatial distribution may not necessarily translate to the UAS traffic.

We use a population density model in this work where the flights endpoints are sampled based on the population density (and hence neither the vehicles locations nor their headings are no longer distributed uniformly) in line with the 'every home aerodrome' vision [31]. As in the basic version of the Dutch model, the flights occupy a single level, so the setup is essentially two-dimensional (this is in line with the current restriction on operations under 400ft [35]; see also [36] for the "horizontal-maneuvers" TCAS work for UTM).

III. Conflict Definition

We consider two UAS A_o (the own UAS, subscript o denotes own) and A_i (the intruder UAS, subscript i denotes intruder), with variables p_o, v_o, u_o and p_i, v_i, u_i respectively having their usual meaning (position, velocity and acceleration control). Positions, velocities, and acceleration are only horizontal. All aircraft remain at the same altitude.

The kinematic motion of the aircraft is given by -

$$\frac{dp_o}{dt} = v_o, \frac{dv_o}{dt} = u_o \tag{2}$$

$$\frac{dp_i}{dt} = v_i, \frac{dv_i}{dt} = u_i \tag{3}$$

$p(t;t_0)$ denotes the solution of the ODE over the time interval $[t_0, t]$ with initial conditions $p(t_0), v(t_0)$. The relative variables with respect to the intruder aircraft are given by -

$$p_r = p_o - p_i,\ \dot{p_r} = \dot{p_o} - \dot{p_i} = v_o - v_i = v_r \tag{4}$$

$$v_r = v_o - v_i,\ \dot{v_r} = \dot{v_o} - \dot{v_i} = u_o - u_i = u_r \tag{5}$$

The equations of motion therefore reduce to -

$$\frac{dp_r}{dt} = v_r, \frac{dv_r}{dt} = u_r \tag{6}$$

A. Conflict with Non Cooperative Aircraft

With Non-cooperative aircraft there is either total lack or uncertainty in the intent information. They will not actively cooperate to resolve a conflict. Then an aircraft is required to assume that the intruder might turn on a dime and head straight over at maximum speed. Hence aircraft are in conflict when they are within the minimum separation D irrespective of whether they are converging or diverging.

Formally, the UAS o and i are in non-cooperative conflict at any time iff $\|p_r\| \leq D$.

B. Conflict with Cooperative Aircraft

Cooperative aircraft are expected to actively cooperate to resolve a conflict with full agreement on each others intent and resolution protocol. Hence, we consider them to be in conflict only if they are within the minimum separation D and on a converging path.

Formally o and i are in conflict at any time iff $\|p_r\| \leq D$ and $\langle p_r, v_r \rangle \leq 0$.

IV. Simulation

To establish our comparison results, we use the simulation setup from [1]. We consider UAS with strictly Vertical Take Off and Landing (VTOL) capability, flying on a fixed flight level as shown in Figure 3. All aircraft are at the same level because with an under 400ft restriction on commercial UAS operations[35], there is very little room for multiple levels. Thus, our setup is two-dimensional – any conflicts between the drones may happen only due to the potential loss of minimum lateral separation.

The airspace is modeled as a cuboidal volume LWH defined by a rectangular area extruded to a given height H. Each UAS is a quadruple (o, d, h, t) i.e. it has an origin, destination, height and start time. A typical flight was shown in figure 3. Each UAS is defined as a Matlab class with properties that include the start time, origin, destination and so on. The flights' origins and destinations were generated randomly based on the population density over the rectangular area. This preserves the actual shape of the geographical area and the volume of airspace used.

The total number n of flights expected during the day was given, and the intensity of the traffic starting or ending at a point p of the domain was proportional to the population density at p (that is, the starting times of the flights from p form a Poisson process with the rate proportional to the density).

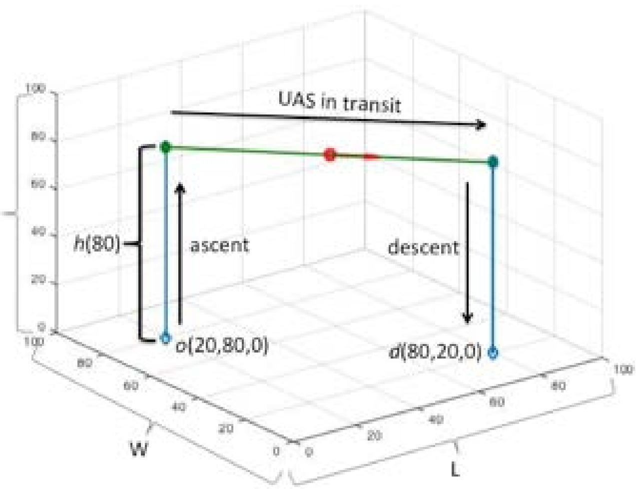

Fig. 3: A typical UAS flight path

We ran simulations for two regions: Bay Area in the US and Norrköping municipality in Sweden (Figure 4). In each of the regions, we simulated six days (72 hours) of traffic, varying n from 10 to 200,000 flights a day and the minimum lateral separation D from 5m to 300m. All UAS travel at a uniform speed from origin to destination. We used the size of the largest component observed at any time as the Metric M to compute capacity with an acceptable limit M'=3.

V. Results

Our simulation data spans traffic densities varying from 100 to 200,000 aircraft per day for conflict distances varying from 5m to 300m. We present our results with a focus on quantifying the impact of cooperation on air traffic complexity and hence airspace capacity.

[1] ran simulations for Bay Area for the non cooperative case with minimum separation values of 50, 100, 150, 200, 250 and 300 meters, with 50m simulation producing the slowest complexity growth and 300m simulation the fastest. It further presented the 99th percentile and largest clusters produced at these separation distances. Hence, for our complexity comparison, we compare those results for the Bay Area with the same ones produced for the cooperative aircraft case. For the airspace capacity comparison, we use the baseline in [39] which produced capacity results for the non cooperative case for Bay Area and Norrköping varying the minimum separation from 5m to 300m and compare it with the same ones produced here for the two metropolitan regions for the cooperative case.

We start by plotting the NTSC versus traffic density curves (figure 5 & 6) for 6 conflict distances - 50m, 100, 150m, 200m, 250m and 300m, to compare our results against those produced for non-cooperative aircraft in [1]. NTSC lies between 0 and 1 due to normalization. It is expected to look like an S-curve but only the bottom part of the curve is attained at the traffic densities we simulated. For Bay Area, we find that with non-cooperative aircraft, the normalized time spent in

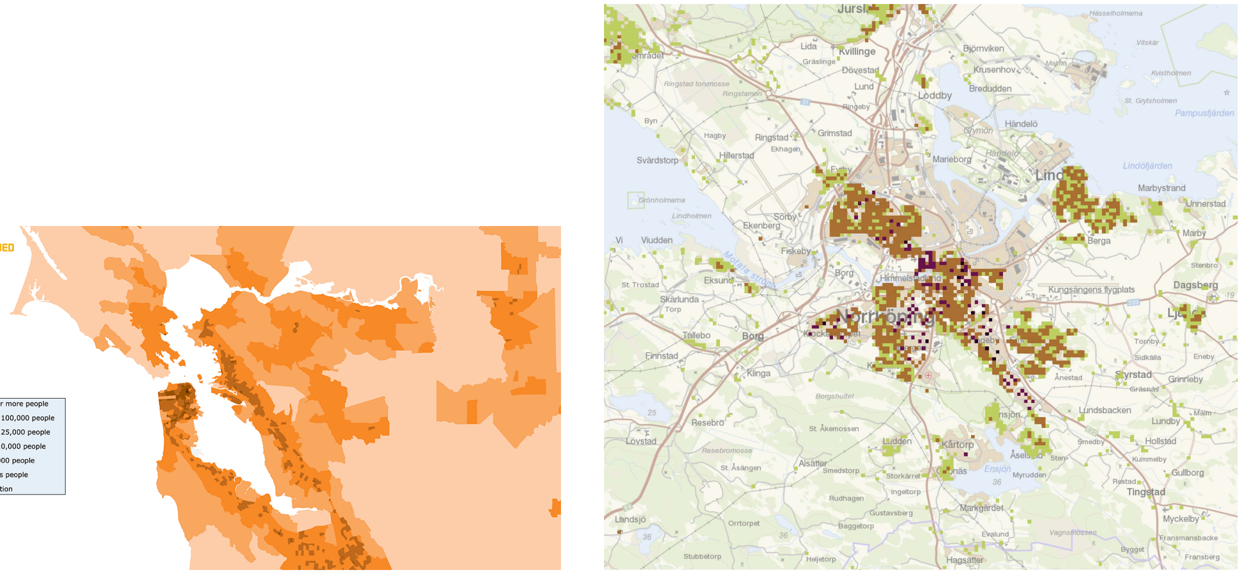

Fig. 4: Population Density Map. Left: the Bay Area [37]. Right: Norrköping municipality [38].

conflict starts visibly increasing past 10,000-20,000 flights per day for all conflict distances. Cooperation pushes this complexity regime change to the range of 70,000-90,000 flights per day. This suggests that the expected future UAS traffic of 100,000 flights per day may potentially be enabled by ensuring cooperative maneuvers between aircraft.

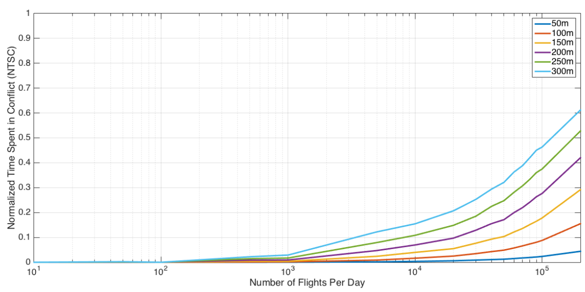

Fig. 5: Growth of NTSC with increasing traffic densities for different conflict distances

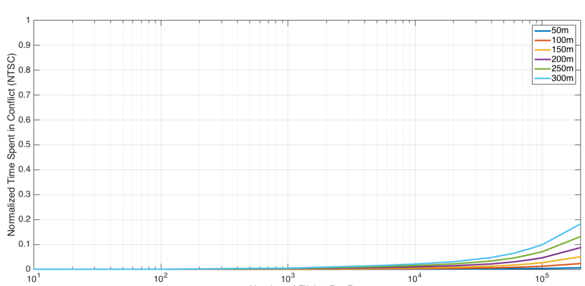

Fig. 6: Growth of NTSC with increasing traffic densities for different conflict distances

This is also suggested by comparing the cluster statistics results from [1] in Table I for non-cooperative scenario with those those for the cooperative scenario in Table II. They list the 99^{th} percentile and largest clusters observed for the above six conflict distances in Bay Area. With 100,000 flights a day, even with a conflict distance of 300m, cooperation reduces the largest clusters to 6 down from 96 and the 99^{th} percentile cluster drops down to 3 aircraft which would be manageable by airborne CDRA systems.

Next, we look at the probability results on the chosen metric (the size of the largest cluster observed). Figures 7, 8 and 9 show the results for Bay Area. Figure 7 shows the probability $\mathsf{P}\{M > M'\}$ of observing large de-confliction problems (clusters of size greater than 3) with increasing traffic density for different conflict distances. For a chosen value of $\eta = 0.2$, we find that the UAS traffic capacity is reached roughly around 30,000 flights per day (between the range 20,000-40,000) in the non co-operative case. Cooperation improves this volume to 80,000 (between 70,000-90,000). Similarly, figures 10, 11 and 12 show the results for Norrköping. The non-cooperative capacity for the same η is attained at around 15,000 (between 10,000-20,000) flights a day while the cooperative capacity is increased to 40,000(between 30,000-50,000). These numbers define the capacities of the airspaces over the respective regions (under flight level assumption as per IV). .

Naturally, it is possible that with the advance of the conflict detection and resolution techniques and hardware, these capacities can be further improved. It is noteworthy that Figures 8, 9, 11 and 12 actually show the entire band of capacity range in light blue and green over all the combinations of the minimum separation and traffic densities chosen. The above mentioned single capacity number are the centre values of the respective capacity ranges. The airspace capacity

TABLE I: Cluster Size Distribution for Non-Cooperative UAS - Bay Area

	(99^{th} Percentile, Largest) Cluster Size					
Flight Rate (per day)	**50m**	**100m**	**150m**	**200m**	**250m**	**300m**
100	(0, 0)	(0, 0)	(0, 0)	(0, 2)	(0, 2)	(0, 2)
1000	(0, 2)	(0, 3)	(0, 2)	(0, 2)	(0, 3)	(2, 3)
10000	(0, 3)	(0, 4)	(2, 4)	(2, 4)	(2, 5)	(2, 6)
100000	(2, 4)	(2, 9)	(3, 13)	(4, 37)	(7, 60)	(13, 96)

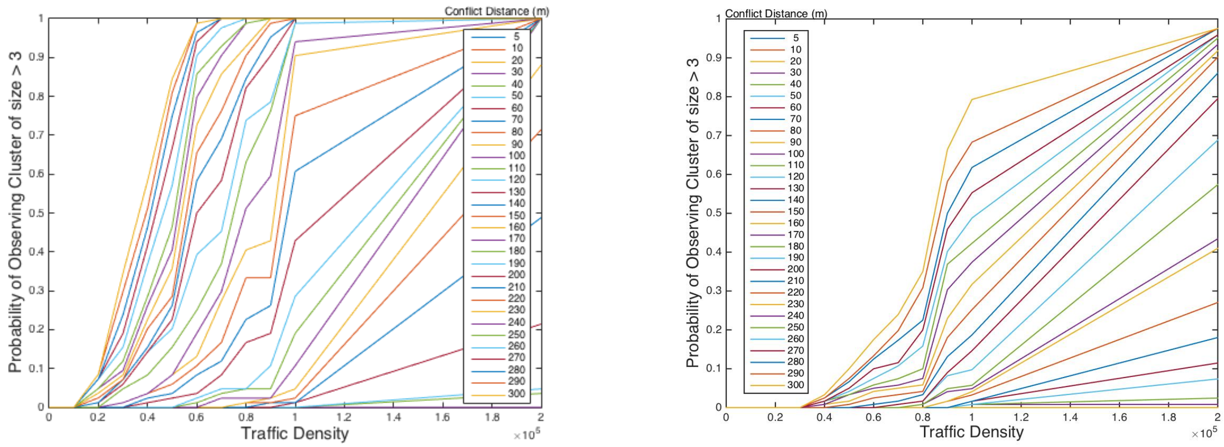

Fig. 7: Bay Area: $\mathsf{P}\{M > M'\}$ as function of N for various D. Left: Non-Cooperative Right: Cooperative.

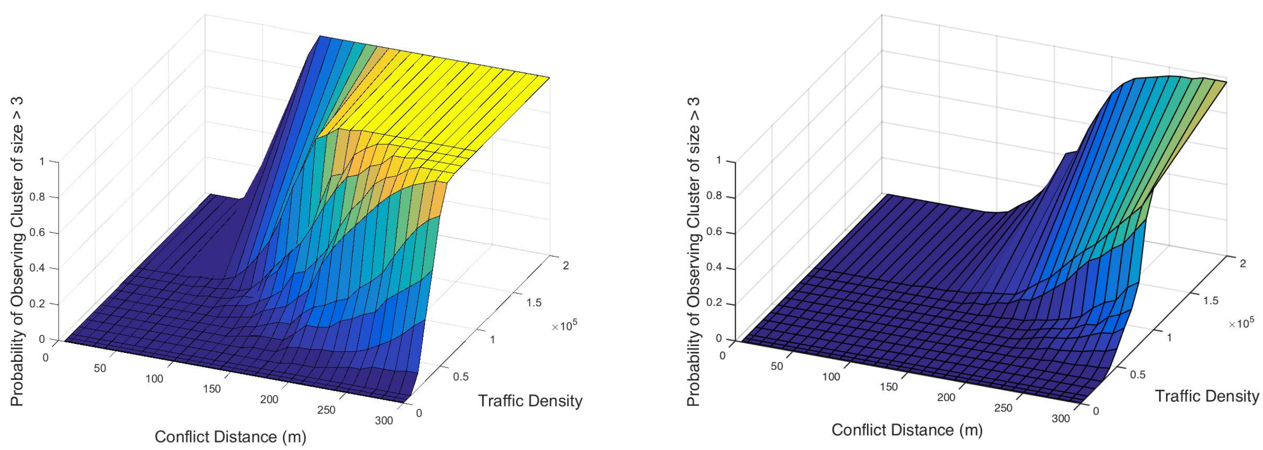

Fig. 8: Bay Area: $\mathsf{P}\{M > M'\}$ as function of both N and D. Left: Non-Cooperative Right: Cooperative.

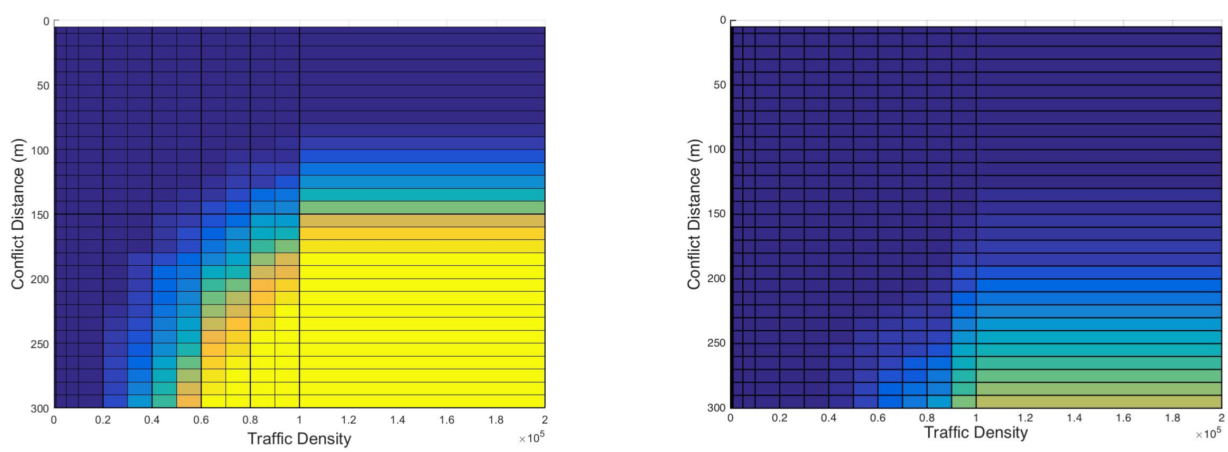

Fig. 9: Bay Area: $\mathsf{P}\{M > M'\}$ as function of both N and D (Top View). Left: Non-Cooperative Right: Cooperative.

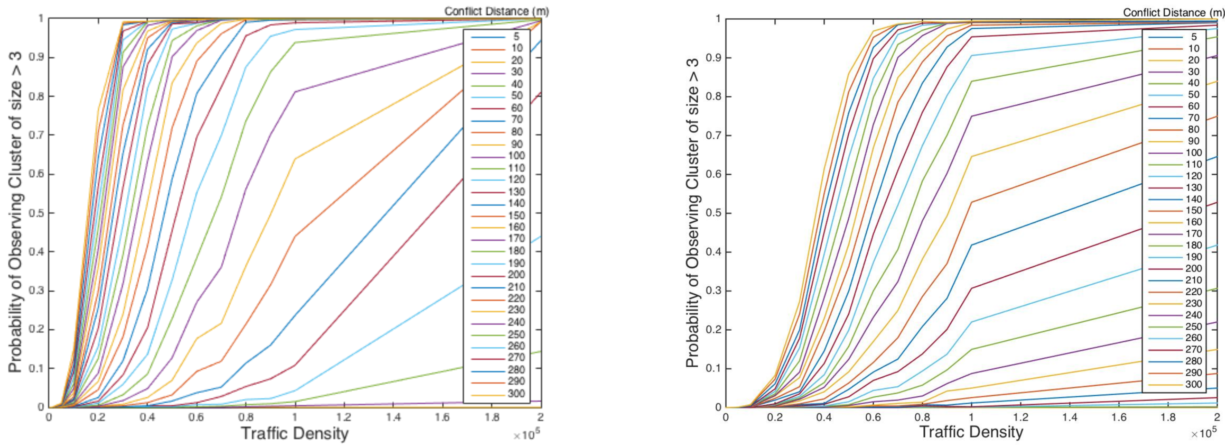

Fig. 10: Norrköping: $\mathsf{P}\{M > M'\}$ as function of N for various D. Left: Non-Cooperative Right: Cooperative.

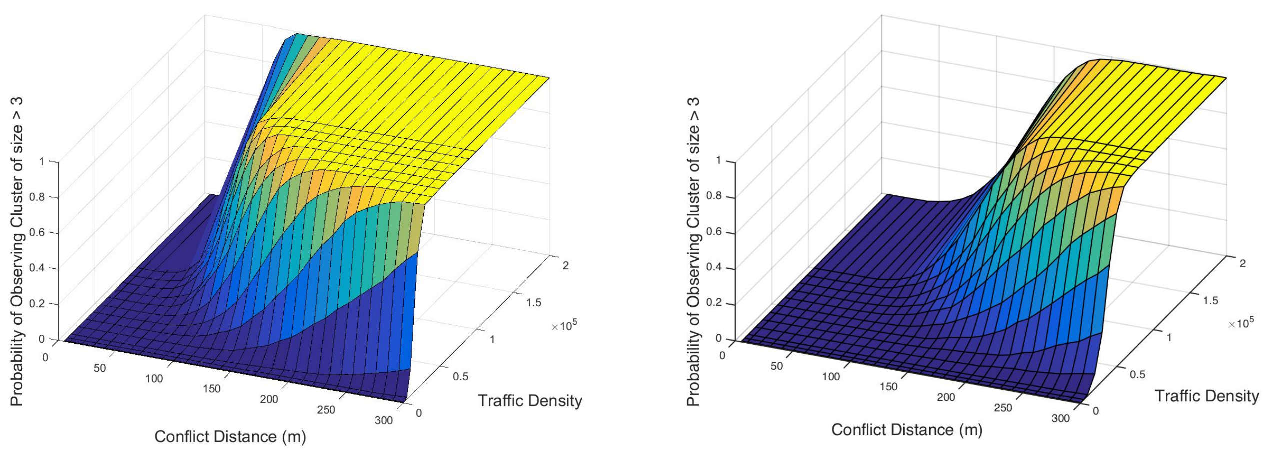

Fig. 11: Norrköping: $\mathsf{P}\{M > M'\}$ as function of both N and D. Left: Non-Cooperative Right: Cooperative.

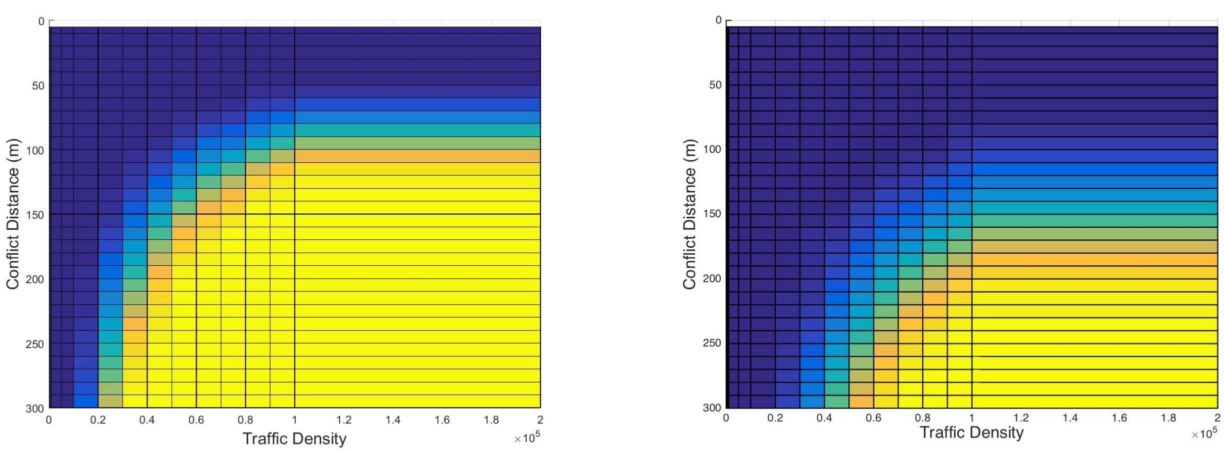

Fig. 12: Norrköping: $\mathsf{P}\{M > M'\}$ as function of both N and D (Top View). Left: Non-Cooperative Right: Cooperative.

TABLE II: Cluster Size Distribution for Cooperative UAS - Bay Area

	(99^{th} Percentile, Largest) Cluster Size					
Flight Rate (per day)	**50m**	**100m**	**150m**	**200m**	**250m**	**300m**
100	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
1000	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(2, 2)
10000	(0, 2)	(0, 2)	(2, 2)	(2, 3)	(2, 3)	(2, 3)
100000	(2, 3)	(2, 4)	(3, 4)	(3, 5)	(3, 5)	(3, 6)

in general increases with decreasing minimum separation tolerance and with cooperation between aircraft for resolving conflicts.

VI. Conclusions and Future Work

We described an analytical process to determine how much UAS traffic can be accommodated in the low altitude airspace over metropolitan regions. Our results show that cooperation between aircraft greatly improves the UAS traffic volumes. The probability of observing large de-confliction problems exhibits thresholds at the airspace capacity. Modes of operations with (N, D) in the blue areas on Figure 8 and Figure 11 are very unlikely to exceed the capacity, while operating in the yellow areas will almost surely exceed the capacity leading to large de-confliction problems. The (thick) regions that separate blue from yellow show the relations between the "critical" traffic intensity and conflict radius. We believe that the graphs like these will help the authorities in quantifying the tradeoffs between the allowable density of the UAV traffic (N) and the CDRA capabilities (D) required to meet the operational requirements of safety in the airspace.

Further, we find that cooperation can also greatly reduce computational complexity for airborne CDRA systems by minimizing the sizes of multi-aircraft de-confliction problems. As a result, we observe that a change in traffic density by an order from 10,000 to 100,000 is still manageable.

Finally, we find a cluster based approach to be an appropriate tool to establish capacity results on unmanned traffic. Next, it is of interest to estimate the airspace capacity under a wider range of metrics M and their allowable limits M'. The current work is based on a simple assumption of conflict as loss of minimum separation and doesn't include any conflict avoidance algorithm as part of the simulation. However, using a CDRA technique widely accepted by the aviation community would lead to better bounds on the airspace capacity and we leave this for future exploration.

One of the most important concerns that the UTM community is currently facing is to measure the volume of unmanned aircraft that can be accommodated in the existing airspace based on considerations of system safety, system performance, spectrum required for communication and noise levels. Our analysis method enables us to respond to that concern. Future extensions of this work will also establish capacity bounds addressing that wider variety of operational requirements.

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References

[1] V. Bulusu, R. Sengupta, and Z. Liu, "Unmanned aviation: To be free or not to be free? a complexity based approach," in *7th International Conference on Research in Air Transportation*, 2016.

[2] V. Bulusu, L. Sedov, and V. Polishchuk, "Noise estimation for future large scale small uas operations," in *NOISECON*, 2017, to appear. [Online]. Available: http://unmanned.berkeley.edu/publications

[3] T. Prevot, J. Rios, P. Kopardekar, J. E. Robinson III, M. Johnson, and J. Jung, "Uas traffic management (utm) concept of operations to safely enable low altitude flight operations," in *16th AIAA Aviation Technology, Integration, and Operations Conference*, 2016, p. 3292.

[4] J. K. Kuchar and L. C. Yang, "A review of conflict detection and resolution modeling methods," *IEEE Transactions on intelligent transportation systems*, vol. 1, no. 4, pp. 179–189, 2000.

[5] R. H. Mogford, J. Guttman, S. Morrow, and P. Kopardekar, "The complexity construct in air traffic control: A review and synthesis of the literature." DTIC Document, Tech. Rep., 1995.

[6] P. Kopardekar, "Dynamic density: A review of proposed variables," *FAA internal document. overall conclusions and recommendations, Federal Aviation Administration*, 2000.

[7] P. H. Kopardekar, A. Schwartz, S. Magyarits, and J. Rhodes, "Airspace complexity measurement: An air traffic control simulation analysis," *International Journal of Industrial Engineering: Theory, Applications and Practice*, vol. 16, no. 1, pp. 61–70, 2009.

[8] E. Sunil, J. Hoekstra, J. Ellerbroek, F. Bussink, D. Nieuwenhuisen, A. Vidosavljevic, and S. Kern, "Metropolis: Relating airspace structure and capacity for extreme traffic densities," in *ATM seminar 2015, 11th USA/EUROPE Air Traffic Management R&D Seminar*, 2015.

[9] G. Granger and N. Durand, "A traffic complexity approach through cluster analysis," in *ATM 2003, 5th USA/Europe Air Traffic Management Research and Development Seminar*, 2003.

[10] K. D. Bilimoria and H. Q. Lee, "Analysis of aircraft clusters to measure sector-independent airspace congestion," in *5th Aviation, Technology, Integration, and Operations Conference*, 2005.

[11] A. Goel, S. Rai, and B. Krishnamachari, "Sharp thresholds for monotone properties in random geometric graphs," in *Proceedings of the thirty-sixth annual ACM symposium on Theory of computing*. ACM, 2004, pp. 580–586.

[12] R. Y. Gazit and J. D. Powell, "Aircraft collision avoidance based on gps position broadcasts," in *Digital Avionics Systems Conference, 1996., 15th AIAA/IEEE*. IEEE, 1996, pp. 393–399.

[13] J. Krozel, T. Mueller, and G. Hunter, "Free flight conflict detection and resolution analysis," in *AIAA Guidance, Navigation, and Control Conf., Paper*, 1996, pp. 96–3763.

[14] J.-W. Park, H.-D. Oh, and M.-J. Tahk, "Uav collision avoidance based on geometric approach," in *SICE Annual Conference, 2008*. IEEE, 2008, pp. 2122–2126.

[15] C. Tomlin, G. J. Pappas, and S. Sastry, "Conflict resolution for air traffic management: A study in multiagent hybrid systems," *IEEE Transactions on automatic control*, vol. 43, no. 4, pp. 509–521, 1998.

[16] T. Schouwenaars, B. De Moor, E. Feron, and J. How, "Mixed integer programming for multi-vehicle path planning," in *European control conference*, vol. 1, 2001, pp. 2603–2608.

[17] K.-Y. Kim, J.-W. Park, and M.-J. Tahk, "Uav collision avoidance using probabilistic method in 3-d," in *Control, Automation and Systems, 2007. ICCAS'07. International Conference on*. IEEE, 2007, pp. 826–829.

[18] M. J. Kochenderfer and J. Chryssanthacopoulos, "Robust airborne collision avoidance through dynamic programming," *Massachusetts Institute of Technology, Lincoln Laboratory, Project Report ATC-371*, 2011.

[19] D. Mellinger, A. Kushleyev, and V. Kumar, "Mixed-integer quadratic program trajectory generation for heterogeneous quadrotor teams," in *Robotics and Automation (ICRA), 2012 IEEE International Conference on*. IEEE, 2012, pp. 477–483.

[20] T. B. Billingsley, L. Espindle, and J. D. Griffith, "Tcas multiple threat encounter analysis," *Massachusetts Institute of Technology, Lincoln Laboratory, Project Report ATC-359*, 2009.

[21] J. Krozel, M. Peters, K. D. Bilimoria, C. Lee, and J. Mitchell, "System performance characteristics of centralized and decentralized air traffic separation strategies," in *Fourth USA/Europe air traffic management research and development seminar*, 2001.

[22] J. M. Hoekstra, R. N. van Gent, and R. C. Ruigrok, "Designing for safety: the 'free flight' air traffic management concept," *Reliability Engineering & System Safety*, vol. 75, no. 2, pp. 215–232, 2002.

[23] S. M. Green, K. Bilimoria, and M. G. Ballin, "Distributed air/ground traffic management for en route flight operations," *Air Traffic Control Quarterly*, vol. 9, no. 4, 2001.

[24] J. Krozel, M. Peters, and K. Bilimoria, "A decentralized control strategy for distributed air/ground traffic separation," 2000.

[25] K. D. Bilimoria, K. S. Sheth, H. Q. Lee, and S. R. Grabbe, "Performance evaluation of airborne separation assurance for free flight," *Air Traffic Control Quarterly*, vol. 11, no. 2, pp. 85–102, 2003.

[26] A. Majumdar, W. Ochieng, and J. Polak, "Estimation of european airspace capacity from a model of controller workload," *Journal of Navigation*, vol. 55, no. 03, pp. 381–403, 2002.

[27] A. Majumdar, W. Y. Ochieng, J. Bentham, and M. Richards, "En-route sector capacity estimation methodologies: An international survey," *Journal of Air Transport Management*, vol. 11, no. 6, pp. 375–387, 2005.

[28] A. Klein, L. Cook, B. Wood, and D. Simenauer, "Airspace capacity estimation using flows and weather-impacted traffic index," in *2008 Integrated Communications, Navigation and Surveillance Conference*. IEEE, 2008, pp. 1–12.

[29] J. Krozel, J. Mitchell, V. Polishchuk, and J. Prete, "Airspace capacity estimation with convective weather constraints," in *AIAA Guidance, Navigation, and Control Conference*, 2007.

[30] J. M. Henry and J. M. Suiter, "Traffic collision avoidance system," Jul. 4 2000, uS Patent 6,085,150.

[31] P. Kopardekar, "Unmanned aerial system traffic management system," Talks at Google, 2016.

[32] J. Hoekstra, "Designing for safety: the free flight air traffic management concept," Ph.D. dissertation, National Aerospace Laboratory NLR, Netherlands, 2001, iSBN 90-806343-2-8, also published as NLR TP-2001-313.

[33] M. R. Jardin, "Analytical relationships between conflict counts and air-traffic density," *Journal of guidance, control, and dynamics*, vol. 28, no. 6, pp. 1150–1156, 2005.

[34] J. M. Hoekstra, J. Maas, M. Tra, and E. Sunil, "How do layered airspace design parameters affect airspace capacity and safety?" in *ICRAT2016-7th International Conference on Research in Air Transportation*, 2016.

[35] "Federal Aviation Administration - Unmanned Aircraft Systems," https://www.faa.gov/uas/, 2016.

[36] R. E. Tompa, B. Wulfe, M. P. Owen, and M. J. Kochenderfer, "Collision avoidance for unmanned aircraft using coordination tables," in *Digital Avionics Systems Conference (DASC), 2016 IEEE/AIAA 35th*. IEEE, 2016, pp. 1–9.

[37] "ArcGIS USA Population Density," https://www.arcgis.com/home/index.html, 2016.

[38] "Swedish University of Agricultural Sciences," http://www.slu.se/en/, 2016.

[39] V. Bulusu and V. Polishchuk, "Threshold based capacity estimation for low altitude airspace," in *AIAA Aviation*, 2017, to appear. [Online]. Available: http://weber.itn.liu.se/ valpo40/pages/nr.pdf